Minimization of the UT1 Formal Error Through A Minimization Algorithm

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Introduction

Suppose that we could put our observations anywhere. Where would we put them to minimize UT1? Of course the resulting schedules would be unrealistic, but maybe we could learn something that will help us in writing realistic schedules.
In this poster we report on our approach to this question and also describe scheduling principles to minimize UT1 formal errors. We also report on insight gained into IVS schedules based on tools we developed to answer our initial question. We focused on the IVS INT01 series which uses the Kokee-Wettzell baseline.

Our Approach

We used the Conjugate Gradient method as implemented in Numerical Recipes to minimize the UT1 formal error. This method uses the Fletcher-Reeves-Polak-Ribiere method to minimize a function provided that you know how to calculate the function and its gradient. We developed the following subroutines for use in this algorithm:

σ_{UT1}: We wrote a subroutine to calculate σ_{UT1} given the observations' a) epochs, b) positions in Az-El at Kokee and c) uncertainty in the observations. This routine builds up the normal equation for the six parameters that we usually estimate for the INT01 Intensives. These are: UT1, an atmosphere offset at Kokee and Wettzell, and a clock offset, clock rate and clock second order term at Wettzell. The normal matrix is inverted, and the UT1 formal error is extracted. We verified our results against the UT1 formal error calculated by Solve and by Sked.

σ_{UT1} gradient: We wrote a subroutine to compute the gradient of UT1 with respect to the azimuth and elevation of the observations. The gradient computation was done numerically, e.g., as follows for the azimuth partial (with a similar equation for the elevation partial):

$$\frac{\partial \sigma_{UT1}(az_j)}{\partial az_j} \cong \frac{\sigma_{UT1}\left(az_j + \frac{\delta az}{2}\right) - \sigma_{UT1}\left(az_j - \frac{\delta az}{2}\right)}{\delta az}$$

Given a set of initial observations, the Conjugate Gradient method uses the above subroutines to move the observations in such a manner that it will minimize UT1. In our implementation of this algorithm, we made two simplifying assumptions: 1) the observation epochs do not change and 2) the observation errors are either a) 30 ps or b) elevation dependent, defined as follows:

$$\sigma_{obs}^2 = (30ps)^2 + \left(\frac{6ps}{\sin(el_{kk})}\right)^2 + \left(\frac{6ps}{\sin(el_{wz})}\right)^2$$

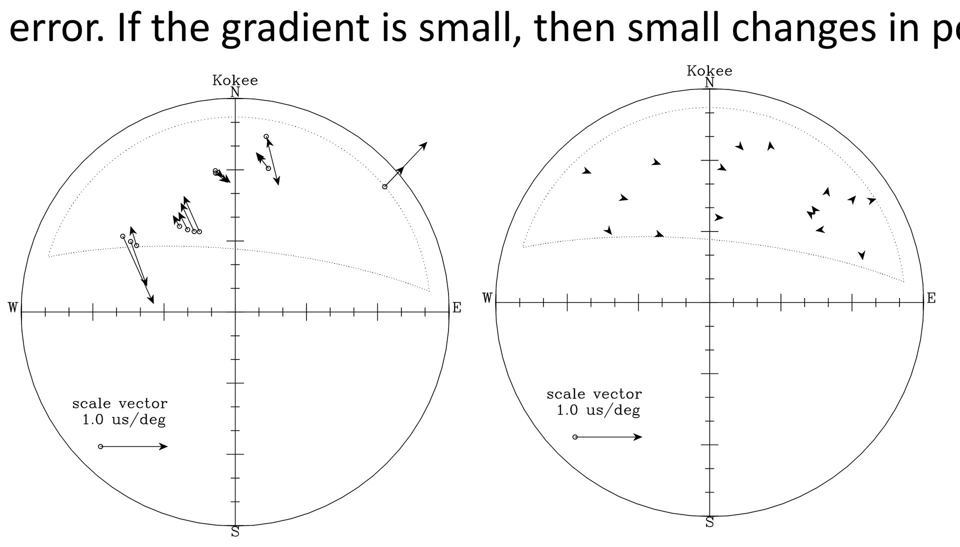
Modifications to the σ_{UT1} subroutine: The algorithm wanted to push the observations below the elevation limit. In order to prevent this, we added a 'penalty term' to the UT1 formal error which was proportional to $(el_{min}+0.1-el)^2$ for $el < el_{min} + 0.1$.

Using Gradients to Evaluate Intensive Schedules

The gradient tells us the sensitivity of UT1 to small changes in the position of the observations:

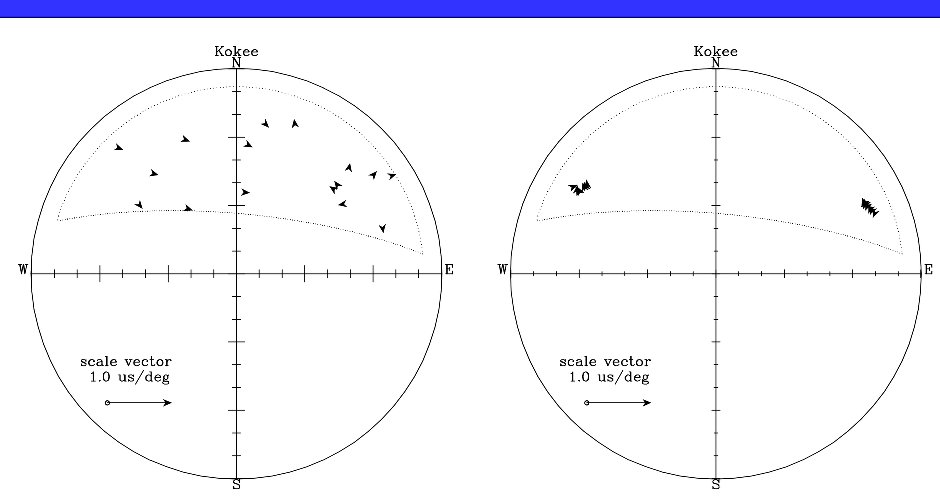
$$\sigma_{UT1}(\overrightarrow{az} + \overrightarrow{\delta az}, \overrightarrow{el} + \overrightarrow{\delta el}) \cong \sigma_{UT1}(\overrightarrow{az}, \overrightarrow{el}) + \frac{\partial \sigma_{UT1}(\overrightarrow{az}, \overrightarrow{el})}{\partial \overrightarrow{az}} \cdot \overrightarrow{\delta az} + \frac{\partial \sigma_{UT1}(\overrightarrow{az}, \overrightarrow{el})}{\partial \overrightarrow{el}} \cdot \overrightarrow{\delta el}$$

If the gradient is large, then small changes in position will result in large changes in the UT1 formal error. If the gradient is small, then small changes in position will have little effect.

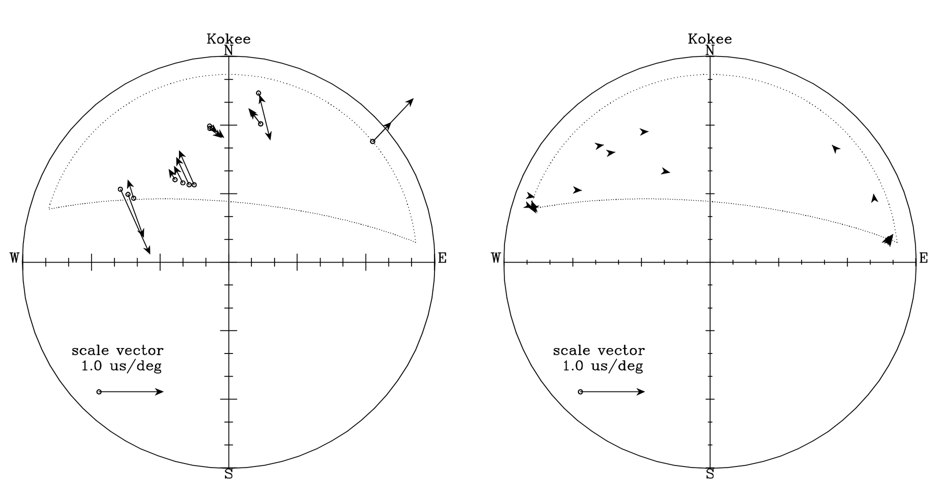


On the left, σ_{UT1} is very sensitive to the changes in positions of a few of the observations. Moving these a few degrees would reduce σ_{UT1} by a few µs. The session's gradient average is 0.436 µs/deg. On the right, σ_{UT1} is relatively insensitive. Moving any of the observations a small amount will have a negligible effect on σ_{UT1}. The session's gradient average is 0.025 µs/deg.

Minimization Results



	Before	After
Formal Error:	9.12 µs	4.98 µs
Avg. Gradient:	0.025 µs/deg	0.002 µs/deg



	Before	After
Formal Error:	24.31 µs	3.78 µs
Avg. Gradient:	0.436 µs/deg	0.004 µs/deg

Before   After

- Elevation dependent sigmas (top row) – observations move towards the corners of the crescent shaped area of mutual visibility between Kokee and Wettzell, but to a cluster at ~ 20 degrees of elevation, in most cases totally, and always with at most a few outlying observations. Gradients are very small.
- 30 ps observation sigmas (bottom row) – observations tend to move to the corners of the area of mutual visibility, although imperfectly. Gradients are a little larger than in the elevation dependent case.

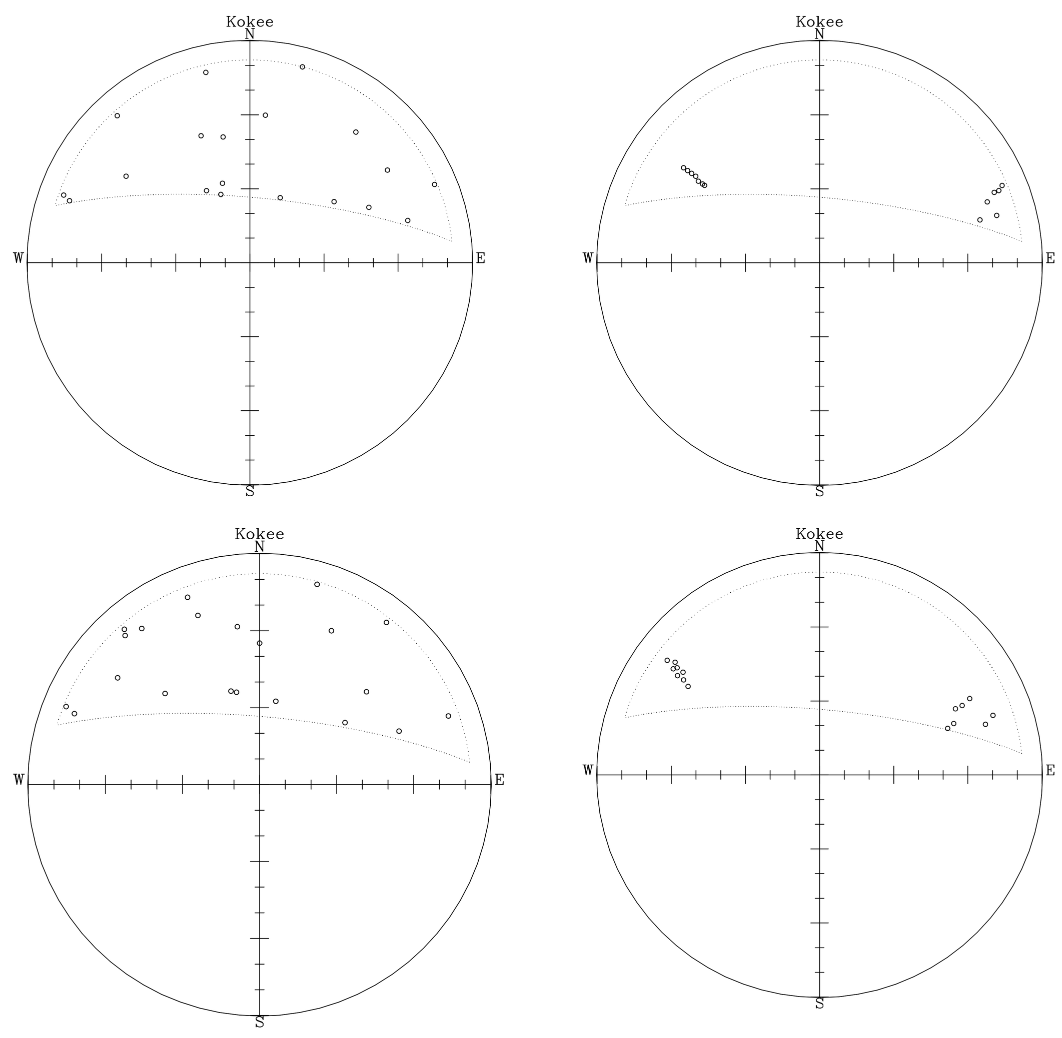
The figures show the results of minimization. In the examples shown, the minimization reduced the UT1 formal errors by ~ 5/6 in an extreme case (bottom) and ~ 1/2 in a normal case (top). Gradients were reduced by ~ 99% and 90%, respectively.

Scheduling Application

Approach: We made manual schedules that used two new approaches suggested by the minimization results and compared them to the operational MSS (formerly USS) observing strategy. This is the strategy in which Sked has available all sources that are mutually visible on the Kokee-Wettzell baseline and tries to make schedules with good sky coverage. We made schedules for October 1, a time of the year with fewer strong sources, and December 16, a more normal time of the year.

- **Cluster approach:** We tried to observe from 66 to 75 degrees azimuth and 301 to 310 degrees azimuth, near 20 degrees elevation. This is analogous to the results from the elevation dependent observation sigma minimization case.
- **Cone:** We tried to observe as close to 90 or 270 degrees azimuth as possible, going no further than 45 or 315 degrees azimuth. We used no elevation restrictions. This is analogous to the 30 ps observation sigma minimization case.

I. Cluster (note: UT1 formal errors were calculated using elevation dependent observation sigmas)



MSS Oct 1
7.89 µs

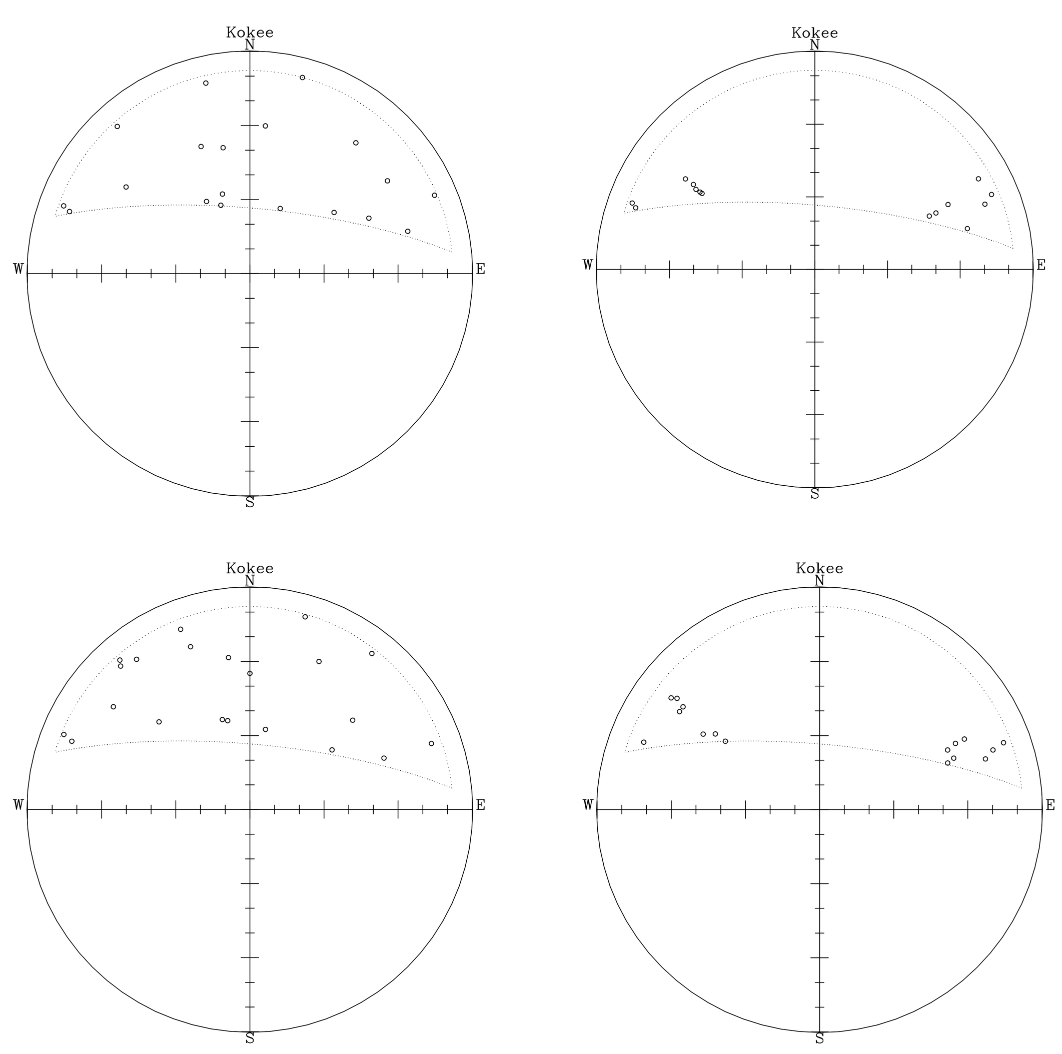
Cluster Oct 1
17.71 µs

MSS Dec 16
8.18 µs

Cluster Dec 16
6.91 µs

The target scheduling area is too small, and it is too hard to find sources near that area, especially for October 1. Sources must be repeated too frequently. This approach does not seem viable.

II. Cone (note: UT1 formal errors were calculated using 30 ps observation sigmas)



MSS Oct 1
5.20 µs

Cone Oct 1
4.37 µs

MSS Dec 16
5.90 µs

Cone Dec 16
3.93 µs

The target scheduling area is much larger, and it is easier to find sources there. This lessens the repetition of sources. This approach is potentially viable, depending on further testing.

Conclusions

- Conjugate gradient minimization identifies sources near the "corners" of mutual visibility as best for minimizing σ_{UT1}. Elevation dependent observation sigmas identify positions near a small area of the sky at ~ 20 degrees elevation, and 30 ps sigmas identify positions nearer the horizon and over a wider part of the sky.
- Due to limited source availability, achieving the elevation dependent positions is not viable in realistic schedules, but achieving the 30 ps positions seems possible.
- Uunila et al., 2013 has said that corner sources are important for obtaining small UT1 formal errors. We go one step further and conclude that, using minimization of the UT1 formal error as the only criterion, only observations at the corners of the mutually visible sky are necessary. (Other scheduling goals and criteria might make it necessary to observe central areas.)

References

Baver, K., Gipson, J., Carter, M. S., and Kingham, K., Assessment of the First Use of the Uniform Sky Strategy in Scheduling the Operational IVS-INT01 Sessions, IVS 2012 General Meeting Proceedings, pages 251-255.

Uunila, M., Nothnagel, A., Leek, J., and Kareinen, N. Influence of Source Distribution on UT1 derived from IVS INT1 Sessions, Proceedings of the 21st Meeting of the European VLBI Group for Geodesy and Astrometry, pages 111-115.

22nd Meeting of the European VLBI Group for Geodesy and Astrometry (EVGA)
Ponta Delgada, Azores, Portugal, 17-21 May 2015

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