Using a Kalman filter to regularize the VLBI nutation time series

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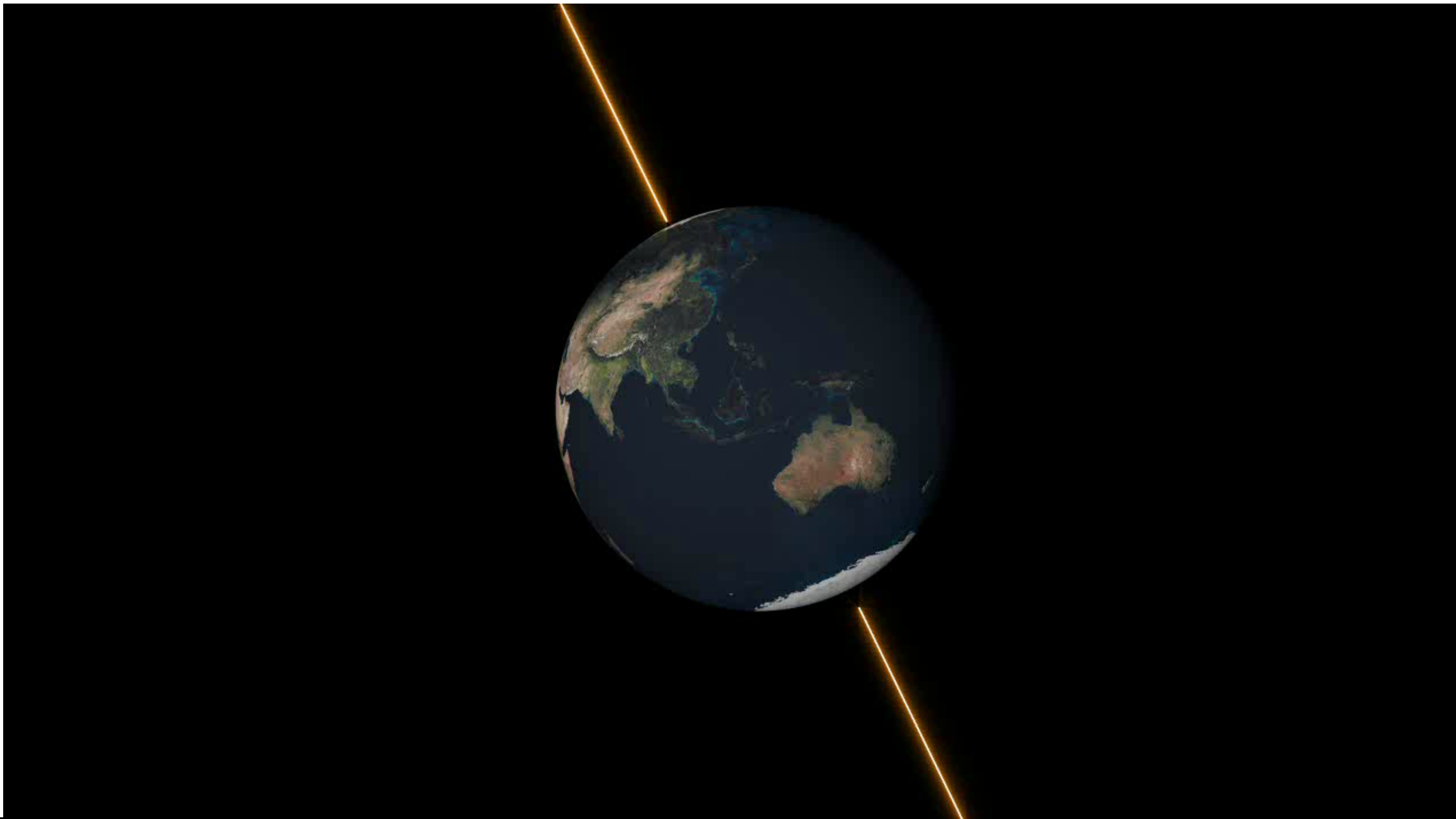
Contents

- Goal: State-of-the-art: VLBI nutation time series.
  - Importance of the VLBI nutation time series;
  - Why regularizing them to daily time series?
- A Kalman filter to regularize the VLBI nutation time series: *nutkal2012.f* .
  - How to use it? Control file and options.
- Applications:
  - Optimal choice of appropriate parameters;
  - Estimator of the quality of the estimate: the Goodness Of Fit.
- Conclusions and perspectives:
  - *nutkal2012.f* for prediction.

What is Nutation, anyway? (nutatio)



Why Regularize?

- Nutation estimated by VLBI at irregular intervals.
- Time-tag depends on epoch of the experiment
- For many purposes nice to have regularly spaced data.
- We (IVS) are closest to the data. We should be able to produce the best results.

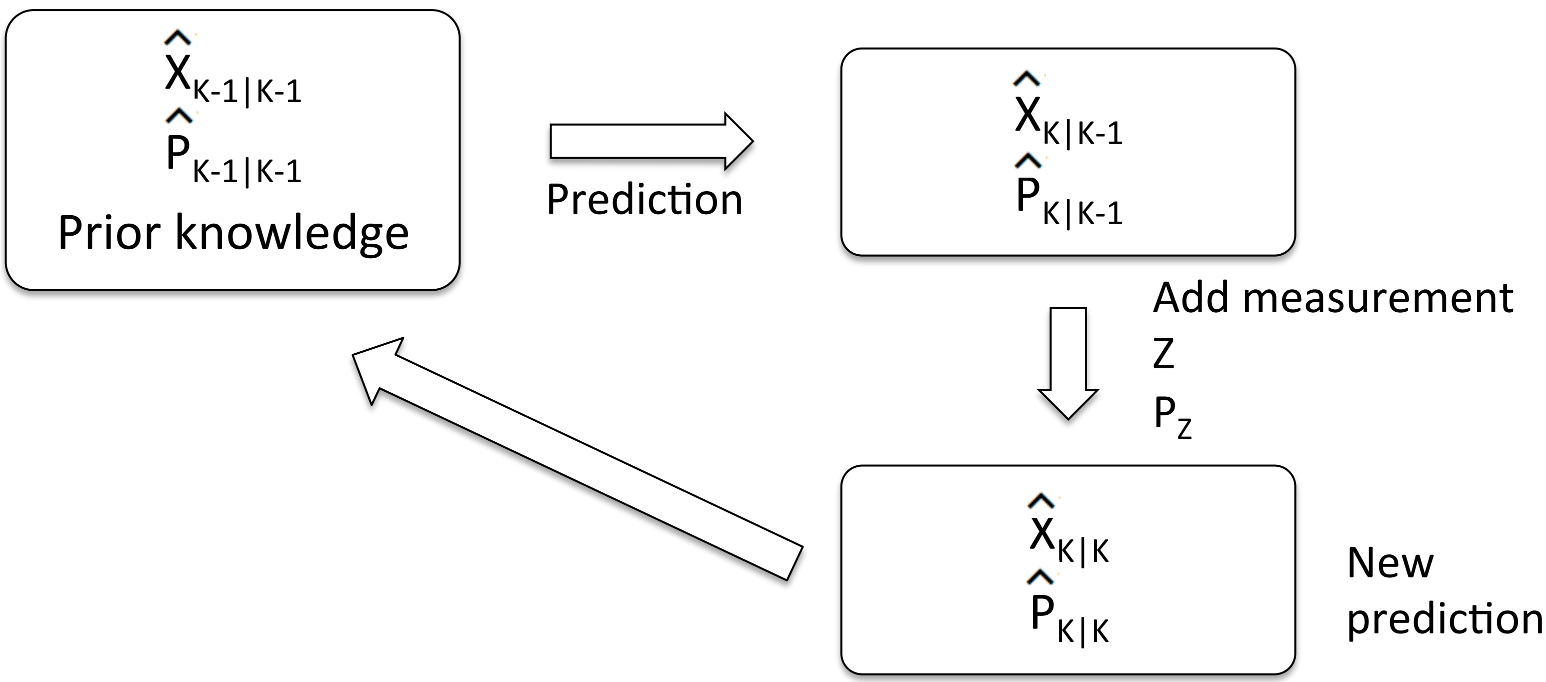
Our Approach

25 years ago we (GSFC) faced a similar problem for UT1 and PM. Needed a good a-priori ERP model. Developed *eopkal* (EOP Kalman filter) still in use.

10 years later developed *nutkal.*

This report is about *nutkal2012. (*An updated version of *nutkal*.)

Kalman filtering - Notions (1)



Kalman filtering - Updating

- Prediction is linear:

$$X_{K|K-1} = AX_{K-1|K-1}$$

$$P_{K|K-1} = AP_{K-1|K-1}A^T + noise$$

The A embodies the physics.

Kalman filtering – Updating Example

- Example: *Linear motion*.
  - State: $\begin{pmatrix} X_{K-1} \\ V_{K-1} \end{pmatrix}$ $P = \begin{pmatrix} \sigma_X^2 & C_{XV} \\ C_X & \sigma_V^2 \end{pmatrix}$ $A = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}$
  - Update

$$\begin{pmatrix} X_{K-1} \\ V_{K-1} \end{pmatrix} \to \begin{pmatrix} X_{K-1} + \Delta t V_{K-1} \\ V_{K-1} \end{pmatrix}$$

$$P \to P = \begin{pmatrix} \sigma_X^2 + 2C_{XV}\Delta t + \Delta t^2 \sigma_V^2 & C_{XV} + \Delta t V \\ C_{XV} + \Delta t V & \sigma_V^2 \end{pmatrix}$$

Kalman filtering – Adding measurement

- Add measurement: Schematically

$$X_{K|K} = \frac{P_{K|K-1}^{-1} X_{K|K-1} + P_Z^{-1} Z}{P_{K|K-1}^{-1} + P_Z^{-1}}$$

Predicted ($X_{K|K-1}$); Measurement (Z)

Note that $P^{-1} \approx \frac{1}{\sigma^2}$: we are just combining according to sigmas.

$$X_{K|K} = X_{K|K-1} - X_{K|K-1} + \left(\frac{P_{K|K-1}^{-1} X_{K|K-1} + P_Z^{-1} Z}{P_{K|K-1}^{-1} + P_Z^{-1}} \right)$$

- Let: $X_{K|K-1} = \frac{(P_{K|K-1}^{-1} + P_Z^{-1}) X_{K|K-1}}{P_{K|K-1}^{-1} + P_Z^{-1}}$. So: $X_{K|K} = X_{K|K-1} + \frac{P_Z^{-1}(Z - X_{K|K-1})}{P_{K|K-1}^{-1} + P_Z^{-1}}$

Prediction ($X_{K|K-1}$); Correction ($\frac{P_Z^{-1}(Z - X_{K|K-1})}{P_{K|K-1}^{-1} + P_Z^{-1}}$)

- Note:

As $P_{K|K-1} \rightarrow \infty$, $P_{K|K-1}^{-1} \rightarrow 0$ give more importance to data.

As $P_{K|K-1} \rightarrow 0$, $P_{K|K-1}^{-1} \rightarrow \infty$ give more importance to prediction.

Kalman filtering - Notions (5)

- Our model for nutation is:
  Integrated Random walk + N harmonic terms.
  Each term has associated with it some noise.
  The harmonics also have associated a width or Q factor.

A Kalman filter to regularize VLBI nutation time series: nutkal2012.f

- The program reads in a series of nutation values from the snoop nutation files…
- Control file.

```
! Nutkal2012  Control file.
! Lines that begin with "!" are comments
! If a ! appears in a line the rest of the line is ignored.
!
! Following control where the data comes from and how much is read in.
Input  snoop.nut                  !input file
Output testVH20b                  !output file
Data_Start      50000.0           !Read in data after this epoch (MJD)
Data_End        0000.d0           ! Not required. If missing or 0.d0  will read in all data.
Max_Psi_sig  1.0                  !Discard data if the sigma is larger than this.
Max_Eps_sig  1.0
! Following controls where the filter is put and the span.
Filter_before     10.0            ! Start filter this much before the first data point
Filter_after      180.0         ! run the filter for This many days after end of data.
Filter_spacing    1.              ! Spacing of output points.
! Can also specify start and end times explicit:
!  Filter_start    date
!  Filter_end      date
!
! The following controls how the signal is modeled.
!  The following control how the signal is modeled.
! Linear  YES/NO  Process_noise
 Linear YES 0.01
! Harmonic Period_days  Width_days Process_noise
  Harmonic 450     0  0.01
  Harmonic 510     0  0.01
  Harmonic 385     0  0.01
```

Optimal choice for the Kalman filter parameters (1)

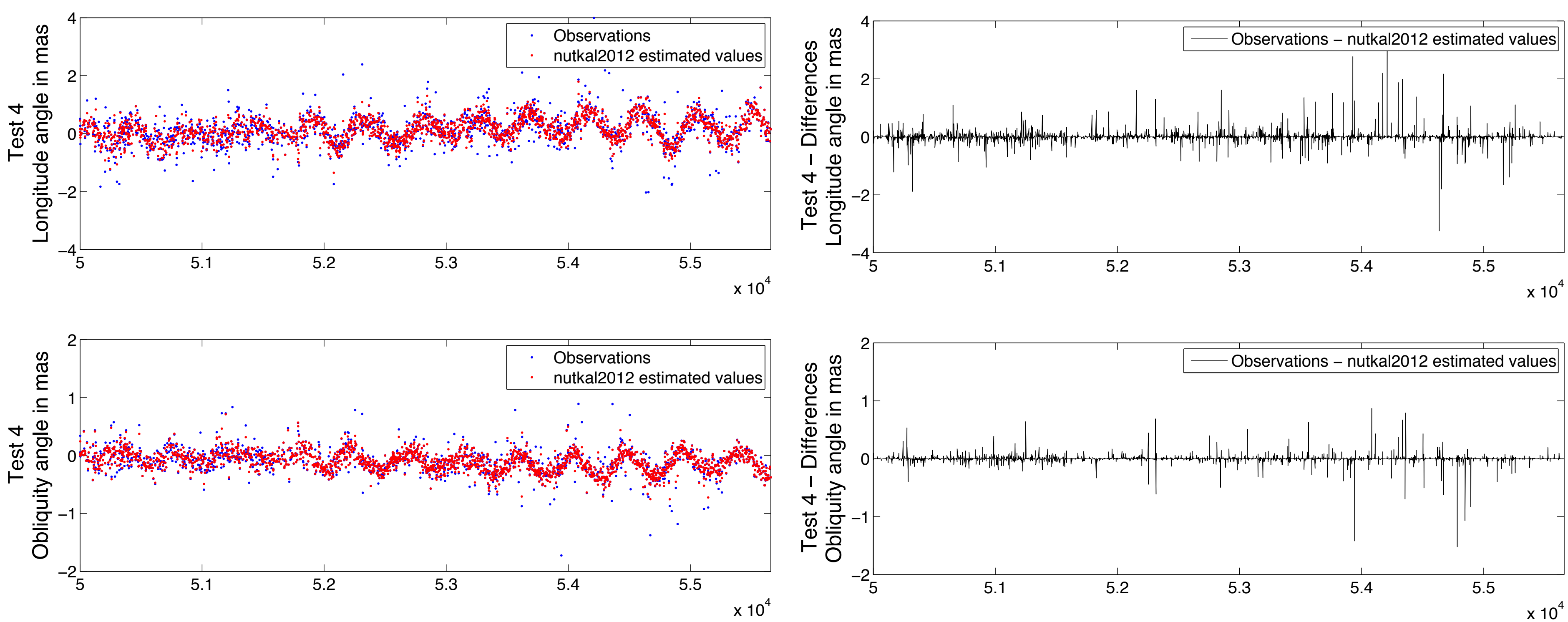
- Snoop file: GSC 2011a solution.
- Which periodic signals?
  - From the FCN (Lambert XX) = -430.21 days;
  - From our study in 2012 (IVS GM) R1 and R4 sessions weekly series = 450, 510 and 385 days;
  - From PSD = 470.25 days.

Optimal choice for the Kalman filter parameters (2)

	Linear		Harmonic			Goodness Of Fit	
	Yes/No	Process noise	Period days	Width days	Process noise	Psi	Epsilon
Test 1	No	-	430	0	0.001	4.140	2.177
Test 2	No	-	470.25	0	0.001	4.140	2.177
Test 3	No	-	470.25	0	0.01	1.968	1.067
Test 4	No	-	450 510 385	0 0 0	0.01 0.01 0.01	**1.387**	**0.744**
Test 5	Yes	0.01	450 510 385	0 0 0	0.01 0.01 0.01	1.282	0.663

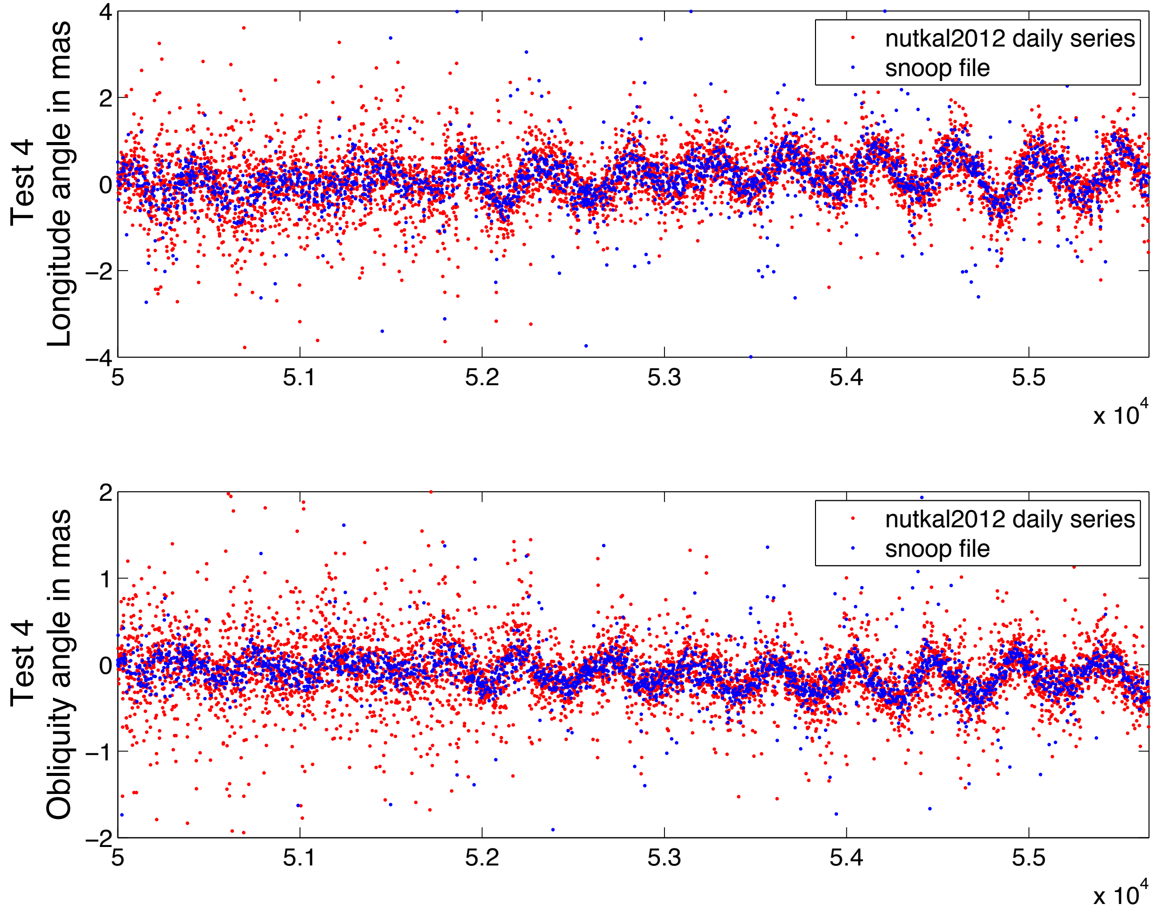
Optimal choice for the Kalman filter parameters (3)

.prn files: observations and estimated values at the date of the observations.



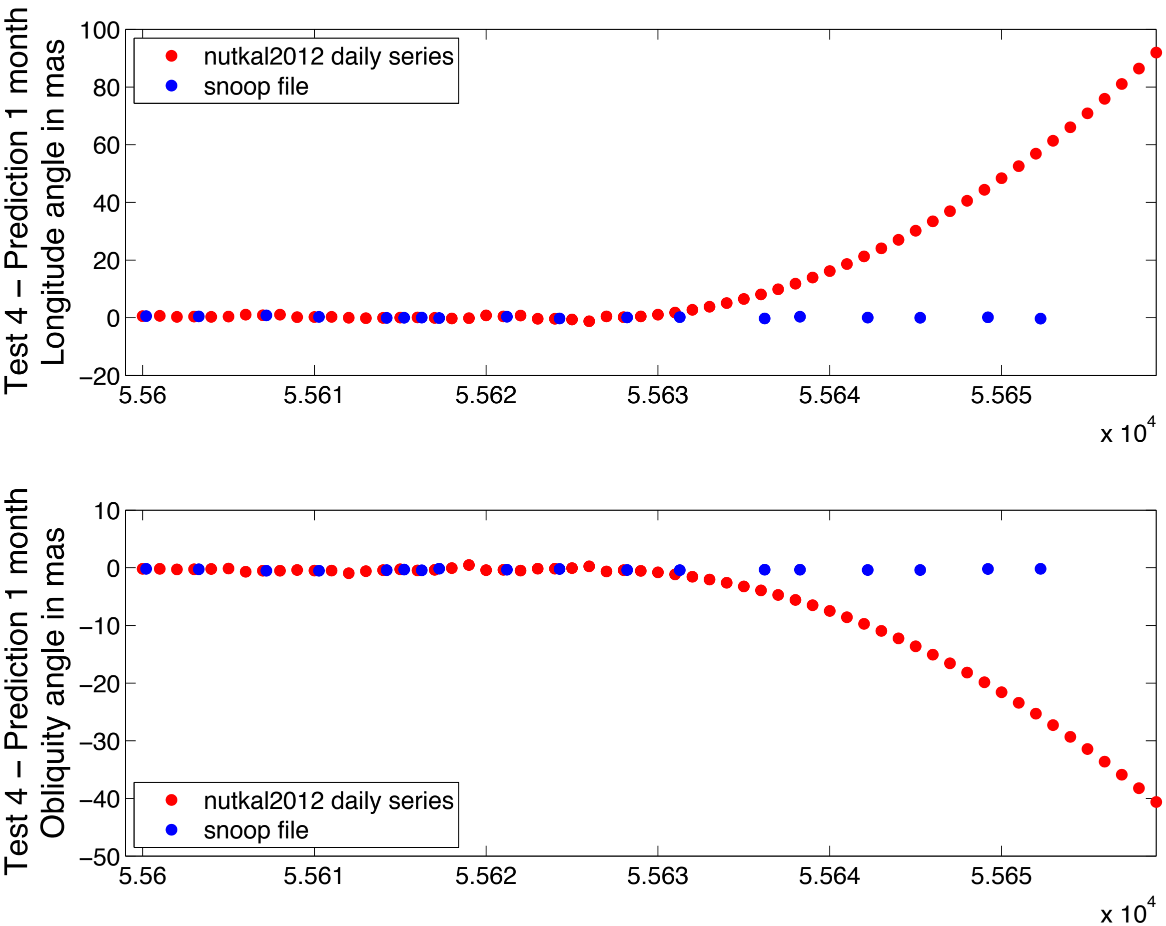
Optimal choice for the Kalman filter parameters (4)

Snoop file and nutkal2012 daily series.



Problem: Filter runs away

Nutkal2012.f
for
prediction



Conclusions and perspectives

Goddard is developing a nutation Kalman filter.

Goal is to use the Kalman filter to regularize the nutation series.

This series can be used:

- As an a priori model for VLBI data analysis.
- For geophysical investigations.

Still a work in progress.